TEST-1 XI (Solution)

1. If $x + \frac{1}{x} = 3$, then the value of $x^3 + \frac{1}{x^3}$ is

(A) 27 (B) 9

(C) 18 (D) 3

1c. If $x + \frac{1}{x} = 3$, then the value ....

यदि $x + \frac{1}{x} = 3$ हो, तो $x^3 + \frac{1}{x^3}$ का ....

Sol. $x + \frac{1}{x} = 3 \Rightarrow \left(x + \frac{1}{x}\right)^3 = 27$

$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 27$

$\Rightarrow x^3 + \frac{1}{x^3} = 27 - 9 = 18$

2. The value of $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots\ldots \frac{1}{\sqrt{120}+\sqrt{121}}$ is

(A) $\sqrt{120} - 1$ (B) 10

(C) $12\sqrt{12}$ (D) 8

2b. The value of $\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \ldots\ldots\ldots\ldots$

$\frac{1}{1+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots\ldots\ldots\ldots$

Sol. $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots\ldots$

$\frac{1}{\sqrt{120}+\sqrt{121}}$

$= \frac{\sqrt{2}-\sqrt{1}}{2-1} + \frac{\sqrt{3}-\sqrt{2}}{3-2} +$

$\frac{\sqrt{4}-\sqrt{3}}{4-3} + \ldots\ldots + \frac{\sqrt{121}-\sqrt{120}}{121-120}$

$= \sqrt{2}-\sqrt{1} + \sqrt{3}-\sqrt{2} + \sqrt{4}-\sqrt{3} + \ldots\ldots +$

$\sqrt{121}-\sqrt{120}$

$= \sqrt{121}-\sqrt{1} = 10$

3. The value of $\dfrac{\sqrt{3-\sqrt{5}}}{\sqrt{2}+\sqrt{7-3\sqrt{5}}}$ is

(A) $\dfrac{3}{\sqrt{5}}$ (B) $\dfrac{1}{\sqrt{5}}$

(C) $\dfrac{\sqrt{5}-1}{\sqrt{5}+1}$ (D) $\dfrac{1}{\sqrt{2}}$

3b. The value of $\dfrac{\sqrt{3-\sqrt{5}}}{\sqrt{2}+\sqrt{7-3\sqrt{5}}}$ ....

$\dfrac{\sqrt{3-\sqrt{5}}}{\sqrt{2}+\sqrt{7-3\sqrt{5}}}$ का ....

Sol. $\dfrac{\sqrt{6-2\sqrt{5}}}{2+\sqrt{14-6\sqrt{5}}} = \dfrac{\sqrt{5}-1}{2+3-\sqrt{5}} = \dfrac{1}{\sqrt{5}}$

4 If $x = \sqrt[3]{7+5\sqrt{2}} - \dfrac{1}{\sqrt[3]{7+5\sqrt{2}}}$, then the value of $x^3 + 3x$ is equal to -

(A) 14 (B) 0

(C) 10 (D) 4

4a. If $x = \sqrt[3]{7+5\sqrt{2}} - \dfrac{1}{\sqrt[3]{7+5\sqrt{2}}}$, then the value

....

Sol. $x = (7+5\sqrt{2})^{1/3} - \dfrac{1}{\left(7+5\sqrt{2}\right)^{1/3}}$

$\Rightarrow x^3 = (7+5\sqrt{2}) - \dfrac{1}{7+5\sqrt{2}} -$

$3\left(7+5\sqrt{2}\right)^{1/3} \dfrac{1}{\left(7+5\sqrt{2}\right)^{1/3}}$

$\left\{\left(7+5\sqrt{2}\right)^{1/3} - \dfrac{1}{\left(7+5\sqrt{2}\right)^{1/3}}\right\}$

$\Rightarrow x^3 = (7+5\sqrt{2}) + (7-5\sqrt{2}) - 3x$

$\Rightarrow x^3 + 3x = 14$

5. If $a + b + c = 1$, $a^2 + b^2 + c^2 = 2$, $a^3 + b^3 + c^3 = 3$, then the value of abc is -

(A) $-\frac{1}{3}$ (B) $\frac{1}{6}$

(C) $\frac{1}{2}$ (D) $\frac{2}{3}$

5b If $a + b + c = 1$, $a^2 + b^2 + c^2 = 2$, $a^3 + b^3 + c^3 = 3$................

$= 2$, $a^3 + b^3 + c^3 = 3$ हो,

....

Sol. $a + b + c = 1 \Rightarrow \quad a^2 + b^2 + c^2 + 2(ab + bc + ca) = 1$

$\Rightarrow ab + bc + ca = -\frac{1}{2}$

$\because a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow 3 - 3abc = (1)\left(2 - \left(-\frac{1}{2}\right)\right)$

$3 - 2 - \frac{1}{2} = 3abc \Rightarrow \quad abc = \frac{1}{6}$

6 Solution of $\log_{(x-1)}(x+2) = \log_{(x+2)}(x-1)$ is .

(A) $\frac{\sqrt{17}+1}{2}$ (B) $\frac{\sqrt{17}-1}{2}$ (C) $\frac{\sqrt{13}-1}{2}$ (D) $\frac{\sqrt{11}-1}{2}$

6c. Solution of $\log_{(x-1)}(x+2) = \log_{(x+2)}(x-1)$ ....

Sol. $\frac{\log(x+2)}{\log(x-1)} = \frac{\log(x-1)}{\log(x+2)}$

$\Rightarrow (\log(x+2))^2 = (\log(x-1))^2 \Rightarrow \log(x+2) = \pm\log(x-1)$

$\Rightarrow (x+2) = (x-1)$ [Not possible]

or $(x+2) = \frac{1}{(x-1)}$

$\Rightarrow x^2 + x - 2 = 1 \quad \Rightarrow x^2 + x - 3 = 0$

$x = \frac{-1 \pm \sqrt{1+12}}{2}$

$x = \frac{-1 \pm \sqrt{13}}{2}$

but to define given question $x > 1$, $x \neq 2$

$\therefore x = \frac{\sqrt{13}-1}{2}$ **Ans.**

7. $\log_{11}\left(1-\frac{1}{3}\right) + \log_{11}\left(1-\frac{1}{4}\right) + \log_{11}\left(1-\frac{1}{5}\right) +........ + \log_{11}\left(1-\frac{1}{242}\right)$ when simplified has the value equal to -

(A) 2 (B) –2 (C) 6 (D) none of these

7b.

Sol. $\log_{11}\frac{2}{3} + \log_{11}\frac{3}{4} + \log_{11}\frac{4}{5} + ... + \log_{11}\frac{241}{242}$

$= \log_{11}\left(\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}..........\times\frac{241}{242}\right) = \log_{11}\left(\frac{2}{242}\right)$

$= \log_{11}\left(\frac{1}{121}\right) = -2$

8. The value of x and y satisfying the equation $(2x - y)\pi + (3x + y) = (6 - y)e + 3$ is (where $x, y \in Q$)

(A) $x = \frac{3}{2}$, $y = \frac{3}{2}$ (B) $x = \frac{3}{2}$, $y = \frac{3}{2}$ (C) $x = \frac{3}{2}$, $y = \frac{3}{5}$ (D) none of these

8d. Since $(2x - y)\pi + (3x + y) = (6 - y)e + 3$

$\Rightarrow 2x - y = 0$

$\Rightarrow 6 - y = 0 \Rightarrow y = 6 \Rightarrow x = 3$

and $3x + y = 3$

No rational value of x and y will satisfy the above equation.

9. $\frac{4}{9^{1/3} - 3^{1/3} + 1}$ is equal to

(A) $(3^{1/3} - 1)$ (B) $(3^{1/3} + 1)$ (C) $(4^{1/3} + 2)$ (D) None of these

9b

Sol. $\frac{4}{9^{1/3} - 3^{1/3} + 1} = \frac{4(3^{1/3} + 1)}{(3^{1/3} + 1)(3^{2/3} - 3^{1/3} + 1)} = 3^{1/3} + 1$

10 If $\frac{1}{\sqrt{2}+\sqrt{3}+\sqrt{5}} = a\sqrt{2} + b\sqrt{3} + c\sqrt{30}$ then the value of a + b + c is (where a, b, c are rational numbers)

(A) 1 (B) 2 (C) $\frac{3}{2}$ (D) $\frac{1}{3}$

10d

Sol. $\frac{1}{(\sqrt{2}+\sqrt{3})+\sqrt{5}} \frac{(\sqrt{2}+\sqrt{3})-\sqrt{5}}{(\sqrt{2}+\sqrt{3})-\sqrt{5}}$

$= \frac{(\sqrt{2}+\sqrt{3})-\sqrt{5}}{2\sqrt{6}}$

$= \frac{1}{12}[2\sqrt{3}+3\sqrt{2}-\sqrt{30}]$

$= a + b + c = \frac{2+3-1}{12} = \frac{1}{3}$

11. If $\log_{10}(x-1)^3 - 3\log_{10}(x-3) = \log_{10} 8$, then $\log_x 625$ has the value equal to :

(1) 5 (2) 4 (3) 3 (4) 2

11b. $\log_{10}(x-1)^3 - \log_{10}(x-3)^3 = \log_{10} 8$

$\Rightarrow \log_{10}\left(\frac{x-1}{x-3}\right)^3 = \log_{10}(2)^3 \Rightarrow \frac{x-1}{x-3} = 2 \quad \Rightarrow \quad x - 1 = 2x - 6$

$\Rightarrow x = 5$ So , $\log_x 625 = \log_5(5)^4 = 4$

12 If A = {2, 3, 4, 8, 10}, B = {3, 4, 5, 10, 12}, C = {4, 5, 6, 12, 14} then $(A \cap B) \cup (A \cap C)$ is equal to
(1) {3, 4, 10} (2) {2, 8, 10} (3) {4, 5, 6} (4) {3, 5, 14}

12a $A \cap B = \{2, 3, 4, 8, 10\} \cap \{3, 4, 5, 10, 12\}$
$= \{3, 4, 10\}$
$A \cap C = \{4\}$
$\therefore \quad (A \cap B) \cup (A \cap C) = \{3, 4, 10\}$

13 Let $N = \dfrac{4^5+4^5+4^5+4^5}{3^5+3^5+3^5} \cdot \dfrac{6^5+6^5+6^5+6^5+6^5+6^5}{2^5+2^5}$ then the value of $\log_2 N$ =

(1) 10 (2) 11 (3) 12 (4) 14

13c. $N = \dfrac{4^5+4^5+4^5+4^5}{3^5+3^5+3^5} \cdot \dfrac{6^5+6^5+6^5+6^5+6^5+6^5}{2^5+2^5}$

$= N = \dfrac{4\times4^5}{3\times3^5}\times\dfrac{6\times6^5}{2\times2^5}$

$= \dfrac{4^6\times6^5}{(2\times3)^5} = \dfrac{4^6\times6^5}{6^5}$

$N = 4^6$

$N = (2^2)^6$

$N = 2^{12}$

$\log_2 N = \log_2 2^{12} = 12$

14 If $\log_4(x+1)+\log_4(x+8) = \dfrac{3}{2}$, then value(s) of x is/are

यदि $\log_4(x+1)+\log_4(x+8) = \dfrac{3}{2}$ हो, तो x का मान है–

(1) 1 (2) 4 (3) 0 (4) 2

14c **Sol.** $\log_4(x+1)+\log_4(x+8) = \dfrac{3}{2}$

$x > -1$ and $\log_4(x+1)(x+8) = \dfrac{3}{2}$

$(x+1)(x+8) = 8$

$x^2+9x = 0$

$x = 0, -9$

but $x = -9$ is not possible $\therefore$ $x = 0$

15. Which of these five numbers $\sqrt{\pi^2}$, $\sqrt[3]{0.8}$, $\sqrt[4]{0.00016}$, $\sqrt[3]{-1}$, $\sqrt{(0.09)^{-1}}$, is (are) rational :

(A) none (B) all (C) the first and fourth (D) only fourth and fifth

15d

Sol. $\sqrt{\pi^2}$ = p → irrational

$\sqrt[3]{0.8} = \dfrac{2}{(10)^{\frac{1}{3}}} \rightarrow$ irrational

$\sqrt[4]{0.00016} = \dfrac{(2^2)^{\frac{1}{4}}}{(10^5)^{\frac{1}{4}}} = \dfrac{2}{10\times(10)^{\frac{1}{4}}} \rightarrow$ irrational

$\sqrt[3]{-1} = -1 \rightarrow$ rational

$\sqrt{(0.09)^{-1}} = \dfrac{1}{0.3} \rightarrow$ rational

16 The expression $\left[\sqrt[3]{\sqrt[6]{a^9}}\right]^4 \left[\sqrt[6]{\sqrt[3]{a^9}}\right]^4$ is simplified to

(1) a^{16} (2) a^{12} (3) a^8 (4) a^4

16d

Hint. $a^{9/6\,.\,4/3}\,.\,a^{9/3\,.\,4/6} = a^4$

17 The number of solution of the equation, $\log(-2x) = 2\log(x+1)$ is
(A) zero (B) 1 (C) 2 (D) None of these

17b $\log(-2x) = 2\log(x+1)$

$-2x = x^2 + 2x + 1 \Rightarrow x^2 + 4x + 1 = 0$

$x = \dfrac{-4 \pm \sqrt{16-4}}{2} = \dfrac{-4 \pm 2\sqrt{3}}{2} \Rightarrow -2 \pm \sqrt{3}$

$x = -2 - \sqrt{3}$ and $x = -2 + \sqrt{3}$

Now अब $-2x > 0$ and $x + 1 > 0$

so $x \in (-1, 0)$

so solution is $x = -2 + \sqrt{3}$

18. The quadratic equation $x^2 - 9x + 3 = 0$ has roots α and β. If $x^2 - bx - c = 0$ has roots α^2 and β^2, then (b, c) is

(1) (75, –9) (2) (–75, 9) (3) (– 87, 4) (4) (–87, 9)

18a

Sol. $\alpha + \beta = 9$, $\alpha\beta = 3$ $\alpha^2 + \beta^2 = b$, $\alpha^2\beta^2 = -c$

$81 - 2 \times 3 = b$, $\Rightarrow 9 = -c$ $b = 75$, $c = -9$

Q.19 Saket added up all the even numbers from 1 to 101. Then, from the total he obtained, he subtracted all odd numbers between 0 and 100. The answer he would have obtained is

A. 0 B. 20 C. 30 D. 50

19. **Option D**

Sum of even no. from 1 to 101

Let $S_1 = 2 + 4 + 6 + \ldots\ldots\ldots\ldots + 100$

This is an A.P with common difference = 2

$\therefore T_n = 100 = 2 + (n-1)2$

$= n - 1 = \dfrac{98}{2}$

$n - 1 = 49 \Rightarrow n = 50$

$\therefore S_1 = \dfrac{50}{2}(2 + 100)$

$= 50 \times 51$

$S_1 = 2550$ ………(1)

and Let S_2 = sum of odd no. between 0 & 100

$S_2 = 1 + 3 + 5 + \ldots\ldots\ldots 99$

A.P with Common difference = 2

$T_n = 99 = 1 + (n-1)\,2$

$\Rightarrow n = 50$

$S_2 = \dfrac{50}{2}(1 + 99) = 50 \times 50 = 2500$ ………(2)

According to question

$S_1 - S_2 = 2550 - 2500$

$= 50$

20. Let $y = \cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\ldots\ldots\ldots}}}}$, then the value of y is

(1) $\frac{\sqrt{13}+3}{2}$ (2) $\frac{\sqrt{13}-3}{2}$ (3) $\frac{\sqrt{15}+3}{2}$ (4*) $\frac{\sqrt{15}-3}{2}$

20d $y = \cfrac{1}{2+\cfrac{1}{3+y}} \Rightarrow y = \frac{3+y}{6+2y+1}$

$\Rightarrow 2y^2 + 7y = 3 + y \quad \Rightarrow 2y^2 + 6y - 2 = 0 \quad \Rightarrow y = \frac{-6 \pm \sqrt{36+24}}{4} \Rightarrow y = \frac{\sqrt{15}-3}{2} \quad \because y > 0$

21. $\sqrt{12-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$ simplifies to -

21 4

Sol. $\sqrt{12-\sqrt{3}+\sqrt{3+8\sqrt{7+4\sqrt{3}}}}$

$= \sqrt{12-\sqrt{3}+\sqrt{3+8 \quad (2+\sqrt{3})}}$

$(\because 7+4\sqrt{3} = (2+\sqrt{3})^2)$

$= \sqrt{12-\sqrt{3}+\sqrt{19+8\sqrt{3}}}$

$= \sqrt{12-\sqrt{3}+(4+\sqrt{3})} \quad (\because 19+8\sqrt{3} = (4+\sqrt{3})^2$

$= \sqrt{12+4} = 4$

22 The number of real solutions of the equation $3x^{\log_5 2} - \frac{1}{2^{\log_5 x}} = 4$ is.

12[1]. $3x^{\log_5 2} - \frac{1}{x^{\log_5 2}} = 4 \quad (x > 0)$

$\Rightarrow 3y - \frac{1}{y} = 4 \quad \Rightarrow 3y^2 - 4y - 1 = 0$

$\Rightarrow y = \frac{2 \pm \sqrt{7}}{3} \quad \Rightarrow x^{\log_5 2} = \frac{2 \pm \sqrt{7}}{3}$

$\Rightarrow 2^{\log_5 x} = \frac{2+\sqrt{7}}{3}$ which will give one value of x.

23. The value of $\dfrac{2^{m+3}\cdot 3^{2m-n}\cdot 5^{m+n+3}\cdot 6^{n+1}}{6^{m+1}\cdot 10^{n+3}\cdot 15^{m}}$ is

23[1]

Sol. $\dfrac{2^{m+3}\cdot 3^{2m-n}\cdot 5^{m+n+3}\cdot 6^{n+1}}{6^{m+1}\cdot 10^{n+3}\cdot 15^{m}}$

$= \dfrac{2^{m+3}\cdot 3^{2m-n}\cdot 5^{m+n+3}\cdot 2^{n+1}\cdot 3^{n+1}}{2^{m+1}\cdot 3^{m+1}\cdot 2^{n+3}.\ \ 5^{n+3}\cdot 3^{m}\cdot 5^{m}}$

$= 2^{o}\cdot 3^{o}\cdot 5^{o} = 1$

24. In a group of 500 persons, 300 take tea, 150 take coffee, 250 take cold drink, 90 take tea and coffee, 110 take tea and cold drink, 80 take coffee and cold drink, and 30 take none of the three drinks. Find the number of person who take all the three drinks.

24[50]

Sol. x(A) (tea) = 300
x(B) (coffee) = 150
x(C) (cold drink) = 250
$n(A \cap B) = 90$
$n(A \cap C) = 110$
$n(B \cap C) = 80$
$n(A \cup B \cup C) = 30$
$n(A \cup B \cup C) = 500 - 30 = 470$
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$
$470 = 300 + 150 + 250 - 90 - 110 - 80 + n(A \cap B \cap C)$
$470 = 700 - 280 + n(A \cap B \cap C)$
$750 - 700 = n(A \cap B \cap C)$
$n(A \cap B \cap C) = 50$

Q.25 The number of three digit numbers abc such that $a \times b \times c = 15$ is

Sol. [6]

$a \times b \times c = 15$

$15 = 1 \times 5 \times 3$

$\Rightarrow$ No. of these digits no. $= 3! = 6$

& numbers are

153	31	51
135	513	15

Total = 6